

Exercise 11

Given $f(x) = 2x^2 + 1$ and $g(x) = 3x - 5$, find the following:

- (a) $f(g(2))$
- (b) $f(g(x))$
- (c) $g(f(x))$
- (d) $(g \circ g)(x)$
- (e) $(f \circ f)(-2)$

Solution**Part (a)**

Evaluate $g(2)$.

$$g(2) = 3(2) - 5 = 6 - 5 = 1$$

Now plug this in to the argument of f .

$$\begin{aligned} f(g(2)) &= f(1) \\ &= 2(1)^2 + 1 \\ &= 2(1) + 1 \\ &= 2 + 1 \\ &= 3 \end{aligned}$$

Part (b)

Plug the formula for $g(x)$ into the argument of f .

$$\begin{aligned} f(g(x)) &= 2(3x - 5)^2 + 1 \\ &= 2(9x^2 - 30x + 25) + 1 \\ &= 18x^2 - 60x + 50 + 1 \\ &= 18x^2 - 60x + 51 \end{aligned}$$

Part (c)

Plug the formula for $f(x)$ into the argument of g .

$$\begin{aligned} g(f(x)) &= 3(2x^2 + 1) - 5 \\ &= 6x^2 + 3 - 5 \\ &= 6x^2 - 2 \end{aligned}$$

Part (d)

Evaluate $(g \circ g)(x)$.

$$\begin{aligned}(g \circ g)(x) &= g(g(x)) \\ &= 3(3x - 5) - 5 \\ &= 9x - 15 - 5 \\ &= 9x - 20\end{aligned}$$

Part (e)

Evaluate $(f \circ f)(x)$.

$$\begin{aligned}(f \circ f)(x) &= f(f(x)) \\ &= 2(2x^2 + 1)^2 + 1 \\ &= 2(4x^4 + 4x^2 + 1) + 1 \\ &= 8x^4 + 8x^2 + 2 + 1 \\ &= 8x^4 + 8x^2 + 3\end{aligned}$$

Now plug in $x = -2$.

$$\begin{aligned}(f \circ f)(-2) &= 8(-2)^4 + 8(-2)^2 + 3 \\ &= 8(16) + 8(4) + 3 \\ &= 163\end{aligned}$$